Leibniz on Infinite Numbers, Infinite Wholes, and Composite Substances[1]

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1. Introduction

Leibniz accepts the actual infinite in nature but rejects infinite number. Are his mathematical commitments out of step with his metaphysical ones? It is generally agreed that Leibniz has a viable response to the problem indicated by this question. On Leibniz's view, there are *infinitely many* created substances, but no *infinite number* of them. What it means to say that there are *infinitely many* substances is simply that *there are more than any number we specify, however large*.

Despite the viability of Leibniz's response, in recent scholarship a second, more specific question has been raised: is Leibniz's account of composite substance—a soul united to an infinitely divided body—inconsistent with his rejection of infinite number? Some commentators think that Leibniz faces difficulty responding to this question in light of how he seems to use the impossibility of infinite number to argue against the world soul. Put briefly (and to be developed further below), Leibniz seems to argue that the world cannot have a soul because the world is infinite: if an infinite body were to have a soul, this would entail an infinite number, which is impossible. The problem is that it seems to follow from this line of reasoning that no body can

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have a soul, since in Leibniz's view, all bodies are actually infinitely divided, and thus have infinitely many parts.

In light of this reconstruction of Leibniz's reasoning, it has been suggested that if Leibniz were to rely on the distinction between *infinitely many* and *infinite number* to respond to this more specific problem then, simply by turning the reasoning around, he would also provide a rejoinder to his own argument against a world soul.

In what follows, I will argue, to the contrary, that Leibniz's widely accepted resolution of the first problem can be used to respond to this more specific problem as well. To see this, I will clarify just what is at stake in Leibniz's rejection of infinite number. I will begin by presenting Leibniz's widely accepted response to the tension between accepting the actually infinite and rejecting infinite number. I will then argue that evaluating Leibniz's rejection of infinite number in the context of his argument against the world soul has led to a misunderstanding of the character of Leibniz's rejection of infinite number. I develop a more plausible way to understand Leibniz's rejection of infinite number, on which the more specific problem no longer has any force. Finally, I suggest that, in light of these results, Leibniz's argument against the world soul is not structured in the way that other commentators have supposed.

2. Infinite Number: The General Concern

As indicated, there are two problems that arise from the rejection of infinite number, one general and one more specific. Let us call the worry that Leibniz's acceptance of the actual infinite is straightforwardly incompatible with his rejection of infinite number the *General Concern*. I will begin by formulating the General Concern and Leibniz's solution to it.

For Leibniz, nature is actually infinite. The world contains infinitely many created substances, and bodies are actually infinitely divided into infinitely many parts. However, Leibniz rejects infinite number. Leibniz clearly asserts his commitment to the actual infinite in nature in the following letter to Simon Foucher written around 1690:

> I am so much in favour of the actual infinite [*l'infini actuel*], that, instead of admitting that nature abhors it, as is commonly said, I hold that nature affects it everywhere, in order the better to mark the perfections of its author. So I believe that there is no part of matter [*il n'y a aucune partie de la matière*] which is not, I do not say divisible, but actually divided [*actuellement divisée*]; and consequently the least particle [*la moindre particelle*] must be regarded as a world full of an infinity [*une infinité*] of creatures. (G I, 416)

The actual infinitude of nature is asserted in terms of both the actual division of the parts of matter and the fact that there is an infinity of creatures in every part of matter, no matter how small.[2] In light of this passage and ones like it, one might expect Leibniz to be sympathetic to the notion of an infinite number. However, this is not the case. Leibniz believes he can demonstrate that the very notion of infinite number is contradictory.

A standard demonstration of the impossibility of infinite number, according to Leibniz, is known as Galileo's Paradox. Here is the paradox, in Leibniz's formulation:

> The number of all squares [*numerous omnium quadratorum*] is less than the number of all numbers [*omnium numerorum*], since there are some numbers which are non-square. On the other hand, the number of all squares is equal to the number of all numbers, which I show as follows: there is no number which does not have its own corresponding square, therefore the number of numbers is not greater than the number of squares; on the other hand, every square number has a number as its side: therefore the number of squares is not greater than the number of numbers. Therefore the number of all numbers (square and non-square) will be neither greater than nor less than, but equal to the number of all squares: the whole will be equal to the part, which is absurd. (A VI 4, 550-551; LoC 177) [3]

[2] Leibniz asserts the actual infinity of nature (in the same sense in which this passage asserts it) from as early as 1680. See the short work "Created Things are Actually Infinite" (LoC 234; A VI 4, 266).

[3] This argument is discussed at length in Arthur (1999), Brown (2000), and Arthur (2001). Arthur supports Leibniz's argument, while Brown suggests that Leibniz's commitment to the part-whole axiom results in an equivocation, relying on different senses of "less than" (22). A similar argument has been made by Benardete (1964)*,* 46-47, Levey (1998), 61, and finally, Van Atten (2009), 3.

From the fact that the even numbers can be set up in a one-to-one correspondence with the natural numbers, Leibniz infers the following conclusion:

> I believe it to be the nature of certain notions that they are incapable of perfection and completion [*perfectionis, atque absoluti*], and also of having a greatest of their kind. Number is such a thing. (A VI 4, 551; LoC 179)[4]

According to Leibniz, the *number of all numbers* is equivalent to *the number of unities* and to *the greatest number*.[5] Leibniz typically uses these three terms when discussing *infinite number* rather than the term "infinite number" itself. [6]

Leibniz himself makes it very clear that his rejection of infinite number can be squared with his acceptance of the actual infinite. In the *New Essays* (1704), he writes,

> [i]t is perfectly correct to say that there is an infinity of things [*une infinite des choses*], i.e. that there are always more of them than one can specify. But it is easy to demonstrate that there is no infinite number [*nombre infini*], nor any infinite line or other infinite quantity [*quantité infinite*], if these are taken to be genuine wholes [*veritables Touts*]. The Scholastics were taking that view, or should have been doing so, when they allowed a 'syncategorematic' infinite, as they called it, but not a 'categorematic' one. (NE 157)

So the question is now: how can these two claims be reconciled? In this passage, Leibniz relies on a distinction between the *syncategorematic* and the *categorematic* infinite.

This distinction is originally a semantic distinction concerning the significations of different types of words. It originates in a famous passage in Priscian's work *Institutiones grammatice* (II, 15) in which he identifies two types of word classes: those that have a definite meaning on their own and those—*syncategorema* or *consignificantia*—that must be combined

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[4] Leibniz begins the *Discourse on Metaphysics* with the same observation and contrasts the notions of power, knowledge and benevolence, which are capable of perfection, with that of number and motion, which are not (G IV, 427; AG 35). For a discussion of the difference between notions that are capable of a greatest of their kind and notions that are not, see Nachtomy (2005).

[5] "But the number of all numbers is the same as the number of all unities [*numero omnium unitatum*] (since a new unity added to the preceding ones always makes a new number), and the number of all unities is nothing other than the greatest number [*numero maximo*]" (A VI 4, 552; LoC 179).

[6] Georg Cantor, notably, does not agree with Leibniz that these terms express equivalent notions, as demonstrated in his diagonal argument that the cardinality of the real numbers is uncountable, while that of the natural numbers and rational numbers is countable. See Cantor (1892).

with words from the first class in order to acquire a definite meaning.[7] This distinction guides the majority of medieval thought on the infinite, replacing (to a large extent) Aristotle's treatment of the infinite in terms of the distinction between actual and potential infinities.[8]

In discussions of the infinite, both in Leibniz's philosophy and more broadly, it is customary to characterize the difference between syncategorematic and categorematic uses of "infinite" in the following way:

> There is an infinity of *x*s just in case:
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> (1) *Syncategorematic:* for every finite number *n*, there is some number *k* of *xs* such that *k* exceeds *n*.
>
> *or*
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> (2) *Categorematic:* there is some number *k* of *x*s such that for every finite number *n*, *k* exceeds *n*.[9]

Leibniz's view is that an actual infinite is permitted in the sense of (1), but not (2). Furthermore, in his view, (1) does not commit him to an infinite number. It merely states that for any number we choose, there is a bigger one out there. (2) on the other hand, does express a commitment to an infinite number, i.e. a number bigger than all others. The central idea here is that, in Leibniz's view, there is no *greatest number*.

As I have mentioned, it is generally agreed that Leibniz's view is plausible, and the view itself has been well documented in the literature on this topic.[10] I would like to add the following two observations:

1) Leibniz's syncategorematic understanding of the infinite tells us something about how he understands the relationship between quantity and the natural world: quantity is not a

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[7] Ishiguro (1990) and Levey (2008) provide helpful discussions of the scholastic origins of this distinction in relation to Leibniz's philosophy of the infinite.

[8] That is not to say that Aristotle's distinction disappears. For the most part, though, "potential" and "syncategorematic" on the one hand and "actual" and "categorematic" on the other are treated as equivalent. There are, however, exceptions to this rule. For a detailed treatment of who held which combination of views, see Ariew and Gabbey (2012), 450ff.

[9] This type of formulation is fairly standard in the literature on this topic. See, for example, Arthur (2001), 107. This particular formulation is very close to one given in Levey (2008), 109-110.

[10] Those who defend the coherence of Leibniz's view on this point: Russell (1900), Rescher (1955), Ishiguro (1990), Carlin (1997), Arthur (1999), (2001), Introduction to LoC, Levey (2008).

fundamental metaphysical feature of the world but a feature of our conceptions and descriptions of it. (This will be important below when considering infinite wholes.)

2) Leibniz understands the infinite as essentially *unending*, but also actual. It does not form a *totality*, yet it is *complete* in the sense that nothing is left out. This is crucially different from a merely potential infinite, which consists in the possibility or potential to go further, but which is, at any given stage, merely finite. The actual infinite *does* go further, *is already further* than any value one can specify. In this sense, Leibniz's rejection of an infinite number is in keeping with his acceptance of the actual infinite: if the infinite could be contained by some number, it would be in some way limited, and thus not actually infinite.

3. Infinite Number: The Specific Concern

What I will call the *Specific Concern* requires a slightly more detailed motivation. The Specific Concern, in outline, is this: does Leibniz's account of composite substance—a form (or dominant monad) joined to an infinitely complex body—violate his rejection of infinite number? Now, there may be reasons to ask this question independent of any other aspects of Leibniz's philosophical system. After all, whatever relation holds between a form (or dominant monad) and a body appears to set that body apart from other bodies in the world, *e.g.* blocks of marble, tables chairs—in a word, what Leibniz calls "aggregates". It is not unreasonable to think that this relation might also have an effect on whether this body, which we can now call an *organic body*, i.e. the body of a substance, or the substance itself entails infinite number. But, more often, scholars have motivated the Specific Concern by appealing to how Leibniz argues against a world soul.

The connection between the world soul and infinite number might seem obscure, but many scholars have read Leibniz as arguing in the following way:

1. If the world were to have a soul, then this would entail the existence of an infinite number (since the world is infinite).

2. Infinite number is impossible.
3. Therefore, the world does not (and cannot) have a soul.[11]

If a world soul is ruled out on the grounds that infinite number is impossible, then it seems that the soul of any body will be ruled out in the same way, since in Leibniz's view, all bodies contain infinitely many substances. As a result, it has been suggested that Leibniz's widely accepted response to the General Concern will not work as a response to the Specific Concern; if it did, then it would also provide a refutation of Leibniz's own argument against a world soul.

As mentioned, the Specific Concern is motivated by how Leibniz argues against a world soul.[12] Consider the following passage from Leibniz's *Theodicy*:

> [T]here is an infinite number of creatures in the smallest particle of matter, because of the actual division of the continuum to infinity. And infinity, that is to say, the accumulation of an infinite number of substances, is, properly speaking, not a whole [*un tout*] any more than the infinite number itself, whereof one cannot say whether it is even or uneven. That is just what serves to confute those who make of the world a God, or who think of God as the Soul of the world; for the world or the universe cannot be regarded as an animal or as a substance. (T §195; H 252)

Carlin (1997) is the first to formulate the difficulty. In his view, Leibniz's rejection of a world soul in this passage relies on the impossibility of infinite number. Carlin finds in this passage the claim that *if the world had a soul*, *then there would be an infinite number*. Given Leibniz's rejection of infinite number, it is a short *modus tollens* inference to the impossibility of a world soul. Based on this reading of the passage, Carlin perceives a problem for Leibniz's theory of substance. He asks the following question:

[11] For this reading of Leibniz's argument see Carlin (1997), Brown (1998), (2001), and (2005), Arthur (1999) and (2001).

[12] I will not discuss Leibniz's argument against the world soul in very much detail; I will focus on the way that the context of this argument has affected how scholars understand Leibniz's rejection of infinite number. I believe, however, that Leibniz's argument against the world soul does not rely on Leibniz's rejection of infinite number in the way that most commentators suppose.

> But why, we may ask, should we admit that infinite aggregates, like the world, cannot admit of a soul? After all, organic bodies, according to Leibniz, just are an accumulation of infinitely many substances, yet he clearly thought they had souls (better: dominant monads). (Carlin, 7)

By parity of reasoning, if the world soul entails infinite number, then so does the soul of any organic body. The question is, then, is Leibniz's account of composite substance inconsistent with his rejection of infinite number?[13]

The task of reconciling Leibniz's views amounts to determining a relevant difference between the body of a composite substance and the world-body, such that the former can have a soul without entailing infinite number while the latter cannot. The approach most often found is to distinguish the type of *unity* that an organic body has (or can have) from the type of *unity* that the world has (or can have). This, commentators suggest, accounts for why the world cannot have a soul—on pain of infinite number—but an organic body can and does have a soul without entailing infinite number. In my view, however, no convincing case has yet been made to distinguish the body of a substance from the world in terms of *unity*.

I will examine two such attempts:

(1) **The Carlin-Arthur Solution**: an organic body is an arithmetical unity, but the world is not.[14]

(2) **Brown's Solution**: an organic body is an *unum per se*, but the world is not.[15]

Each view faces difficulties on its own terms. Arthur (2001), developing Carlin (1997)'s view, suggests that the world-body differs from an organic body in that the latter has arithmetical unity,

[13] We might also ask: why would the world soul entail infinite number? According to Carlin et al. the answer is that the world would become a whole, to wit an *infinite whole*, which stands in violation of Leibniz's rejection of infinite number. However, as my arguments will show, this cannot be the correct understanding of Leibniz's argument.

[14] Here is Arthur: "I claim that for Leibniz a body is a whole in distributive mode, although, unlike the universe, its parts can sum to an arithmetical unity….Thus a converging infinite series can be regarded as a whole, since it (more accurately, the sequence of its partial sums) is limited by a finite number. But a diverging series cannot, since it is not limited by any finite number" (Arthur 2001, 110-112).

[15] Here is Brown: "Leibniz's argument against a world soul is precisely the argument that…the world does not possess a soul because it is not one per se" (Brown 2005, 454).

which the former lacks.[16] In Arthur's view, this means that an organic body is like an infinite *converging* numerical series—e.g. the Dichotomy series: ½ + ¼ + …—while the world is like an infinite *diverging* numerical series—e.g. 1 + 2 + … What Arthur is attempting to capture here is rather intuitive: while an organic body has a merely finite magnitude (like the sum of a converging series), the world has an infinite magnitude (like the sum of a diverging series).[17]

Intuitive though it may be, I do not think that the Carlin-Arthur solution succeeds in its aim, i.e. to distinguish the *unity* that an organic body has from the *unity* that the world-body has. The text that Arthur and Carlin rely on, in order to formulate the notion of *arithmetical unity* is the following:

> A fraction of an animal, or a half-animal, therefore, is not one being per se, since this can be understood only of the body of the animal, which is not one being per se but an *aggregate*, and has an *arithmetical unity* and not a *metaphysical unity*.... Matter (that is, secondary matter), or a part of matter, exists in the same manner as a herd or a house, that is, as a *being by aggregation*. (LDB 31; emphasis added)

The contrast Leibniz develops in this passage is the standard distinction between *unum per se* and *unum per accidens*. For Leibniz, as is well known, *per se* unity is intrinsic unity, while *accidental* unity is not intrinsic unity, but arises from some mind grouping a collection of things together.[18] Consequently, Leibniz will often characterize an *ens per accidens* as "semi-mental".[19] There is no indication in this passage that arithmetical unity has any significance other than the standard meaning of *unum per accidens*. And though Leibniz will acknowledge that accidental unity can come in degrees,[20] there is no basis for concluding either that "arithmetical unity" tracks a certain degree of accidental unity or that a difference in degree of accidental unity can

[16] I should note that Arthur suggests a different view about the wholeness of bodies in Arthur (2011). See footnote # below.
[17] Carlin also explicitly links the ability to have a soul with being an arithmetical unity (1997, 12).
[18] G II, 100; Ma 126.
[19] G II, 304; LDB 31.
[20] See the 30 April 1687 letter to Arnauld (G II, 100; Ma 126).

account for why some collections—i.e. organic bodies—can have souls while others—i.e. the world-body—cannot.[21]

Arthur (and, by proxy, Carlin) is on to something, I think, in highlighting that there is a crucial difference between an organic body and the world-body in that the former is *limited*, while the latter is not: an organic body has a finite *size*. However, we can draw no conclusions about the *unity* of the collection of terms from this admittedly plausible observation. The conclusion that we should draw from this is as follows: though the distinction between converging and diverging infinite series appears to model a plausible difference between organic bodies and the world-body, it cannot provide any grounds for attributing one sort of unity to an organic body and another sort of unity to the world-body. Thus, it cannot provide the basis for why the world soul would entail infinite number, but the soul of an organic body does not.

Brown's solution is also developed as an account of the different types of unity that apply to the world-body and an organic body. In Brown's view, an organic body is an *unum per se*, but the world-body is not (and cannot be). However, there is strong evidence that Leibniz would deny that an organic body is an *unum per se*. Though the *substance* to which the matter belongs is certainly one *per se*, the matter of that substance—its body—is not. This is true whether we consider the body as so much matter—i.e., *mass*—or whether we consider it, as Leibniz does, as an aggregate of substances—i.e., second matter.

In my view, Brown's mistake is to identify the s*ubstance* with the *body*. There is good reason both to resist the identification of a substance with a particular body and to deny that the body of a substance is one *per se*. Consider the following passage to Arnauld:

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[21] Leibniz believes that there is an important difference between converging and diverging numerical series. There is a sense in which the former has a sum while the latter does not (see A VI 3, 503; LoC 99). But this does not entail that one has a certain type of unity that the other lacks.

> ...[i]t is the animated substance [*substance animée*] to which the matter [*matiere*] belongs that is truly one being [*veritablement un estre*], and the matter taken as a mass in itself [*prise pour la masse en elle même*] is only a pure phenomenon or well-founded appearance, as also are space and time. (G II, 118; WF 131)[22]

In this passage, Leibniz both distinguishes the *animated substance* from the matter belonging to it, and explicitly denies that the matter belonging to it is truly one being.

In the *New Essays*, Leibniz makes an analogous philosophical point, this time in terms of "organic bodies", which are the bodies belonging to substances. Consider:

> So we must acknowledge that organic bodies as well as others remain 'the same' only in appearance, and not strictly speaking. It is rather like a river whose water is continually changing, or like Theseus's ship which the Athenians were constantly repairing. But as for substances which possess in themselves a genuine, real, substantial unity, and which are capable of actions which can properly be called 'vital'; and as for substantial beings, *quae uno spiritu continentur* as one of the ancient jurists says, meaning that a certain indivisible spirit animates them: one can rightly say that they remain perfectly 'the same individual' in virtue of this soul or spirit which makes the *I* in substances which think. (NE 231)

Once again, we find two important claims being made: "organic bodies" are distinguished from "substances"; and while organic bodies do not persist, and thus cannot be *per se* unities, substances do persist because they have "genuine, real substantial unity".[23]

In my view, then, neither the Carlin-Arthur solution nor Brown's solution works on its own terms. The mistake that both attempts make is that they attempt to distinguish the world-body from an organic body in terms of unity. But there is a more important reason to resist these solutions: they inaccurately construe Leibniz's rejection of infinite number as a result primarily

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[22] One feature of this passage that makes a definitive interpretation difficult is Leibniz's use of "matter" [*masse*] instead of "body". But if the point is being made only about *masse*, then see the passage from the *New Essays* below for Leibniz's view of *second matter* and in particular, *organic bodies*. It is worth noting, though, that Leibniz will sometimes describe second matter as a "mass of monads". See, for example, LDV 438.

[23] Brown tries to account for how an organic body can both be an *unum per se* and have parts (since, as we have seen, Leibniz denies that an *unum per se* has parts). Brown offers a distinction between *wholes in multitude* and *wholes in magnitude*, the latter of which is supposed to be compatible with being an *unum per se* (2005, 469). I do not think Brown's distinction is textually grounded, but it need not concern us in any case, since by virtue of the fact that an organic body (on my reading) is not an *unum per se*, there is no need to reconcile this with the fact that a body has parts.

about unity. I will argue in the next section that there is an important difference, in Leibniz's view, between *wholeness* and *unity*, and that his rejection of infinite number is primarily concerned with *wholeness*, not *unity*.[24]

4. Infinite Wholes

If we look at a sampling of texts in which Leibniz formulates his rejection of infinite number, we see that though the notion of *unity* is present, he expresses his view in terms of *wholeness* as well. Here is a sampling, spanning about thirty years:

> Hence it follows either that in the infinite the whole is not greater than the part, which is the opinion of Galileo and Gregory of St. Vincent, and which I cannot accept; or that infinity itself is nothing, i.e. that it is not one and not a whole [*non esse Unum nec totum*]. (A Vl 3, 168; trans. Arthur 1999, 107)

> I concede an infinite multitude, but this multitude forms [*facit*] neither a number nor one whole [*unum totum*]. It only means that there are more elements than can be designated by a number, just as there is a multitude or complex of all numbers; but this multitude is neither a number nor one whole. (GM III, 575; LoC, lxiii)

> I maintain, strictly speaking, that an infinite composed from parts [*ex partibus constans*] is neither one nor a whole [*neque unum esse, neque totum*], and it is not conceived as a quantity except through a fiction of the mind [*nec nisi per fictionem mentis concipi ut quantitatem*]. The indivisible infinite alone is one, but it is not a whole; that infinite is God. (LDB 53)

> It is perfectly correct to say that there is an infinity of things [*une infinité de choses*], i.e. that there are always more of them than one can specify. But it is easy to demonstrate that there is no infinite number [*nombre infini*], nor any infinite line or other infinite quantity, if these are taken to be genuine wholes [*des veritables Touts*]. (G V, 144; NE 157)

[24] The distinction between wholeness and unity has been noticed by others, but not developed in a satisfactory way. See, e.g., Carlin (1997), 12; Brown (2005), 462ff.

In these texts, we find a collection of different terms in which Leibniz's rejection of infinite number is expressed: *not one and not a whole*, *not one whole*, *neither one nor whole*, *not a genuine whole*. Certainly both unity and wholeness are present in these texts. What is particularly interesting, though, is that, at least in some of the passages, Leibniz clearly distinguishes between the two. From these passages alone, it is not clear just how unity relates to wholeness. But whatever the relationship between them, these texts suggest that they are philosophically distinct notions.

Further insight into how Leibniz understands the relationship between unity and wholeness can be found by examining his pervasive distinction between *substances* and *aggregates*. This distinction runs throughout Leibniz's texts, from very early to very late. Consider the following passage from *Definitiones* (1685):

> If, when several things are posited, by that very fact some unity [*unum*] is immediately understood to be posited, then the former are called *parts* [*partes*], the latter a *whole* [*totum*]. Nor is it even necessary that they exist at the same time, or at the same place; it suffices that they be considered at the same time [*eodem tempore considerentur*]. Thus from all the Roman emperors together, we construct [*conficimus*] one aggregate [*aggregatum*]. But actually no entity that is really one [*Ens vere Unum*] is composed of a plurality of parts, and every substance is indivisible, and those things that have parts are not entities [*Entia*], but merely phenomena (A VI 4, 627; LoC 271).

This passage is quite rich: it both presents the distinction between substances and aggregates and gives a characterization of parts and wholes. According to this passage, it is a sufficient condition of *being a whole* that some collection of things is *considered at the same time*. Notice that the phrase *considered at the same time* indicates that a perceiving mind plays a crucial role in the creation of wholes. For Leibniz, then, *wholes* are products of mental acts. They do not exist unless the parts are *considered together* by some mind at some time.

But precisely in virtue of *being a whole*, the collection of things being considered is not *really an entity*. Thus, there is a stark contrast between wholes and unities: a whole is something with parts; a unity *has no parts*. Leibniz explicitly claims, not once but twice in the passage, that *nothing with a plurality of parts is really an entity*. The view expressed here confirms the view indicated by the various passages above: wholes are radically different from unities.[25] Since wholes are collections of parts, no whole is really one, i.e. a true unity.[26]

It is worth highlighting that this passage is concerned with what we might call *finite wholes*, i.e. collections of finitely many parts. The collection of Roman Emperors, as in the example, though a strange collection in that it is spatio-temporally disjoint, has only finitely many things in it. The view Leibniz expresses in this passage is that not even collections of only finitely many parts are really one, i.e. they do not have true unity. All wholes have merely accidental unity.[27]

The passage from *Definitiones*, taken together with the passages above, provides evidence that Leibniz sees a stark contrast between wholes and unities. But, because it makes these claims about finite wholes, it also gives us something more; it provides a rationale for reading Leibniz's rejection of infinite number as concerned primarily with wholeness. Consider: both finite and infinite wholes are merely accidental unities. It would be trivial, therefore, for Leibniz to deny that an infinite collection is *one* because he denies that even finite collections are *one*.[28] This confirms that we should understand Leibniz's rejection of infinite number as a rejection of infinite wholes in particular.

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[25] This is also noticed by Carlin (1997) and Brown (2005), though they do not fully develop its significance.
[26] It should be noted that this text is from Leibniz's Middle Years. It suggests that even in the Middle Years, Leibniz is committed to the view that substances do not have parts. Though there is still room for some development in Leibniz's view, I do not see any significant development on this particular point.
[27] Carlin (1997) suggests that Leibniz's view about the relationship between wholeness and unity changes around 1700. In Carlin's view, prior to 1700 Leibniz allowed for wholes with true unity, while after this he does not. This suggestion is complicated, however, by the passage from *Definitiones* cited above. See Carlin (1997), 23, footnote 26, and Brown (2005), 463. On this point, I agree with Brown: Leibniz's understanding of "whole" does not change significantly between the 1680s and the end of his life.
[28] This observation is also made by Russell (1900) and Brown (2005), though they do not develop the significance in the same way I do.

5. Wholes versus Fictional Wholes

Not only does Leibniz distinguish between wholes and unities, he also distinguishes between different types of wholes, and as it turns out, this distinction is crucial to properly understanding his rejection of infinite number. Although finite wholes and infinite wholes are alike in that both result from the parts being *considered at the same time* and both are contrasted with unities, there are important reasons to distinguish between them as well. The distinction is rather intuitive, though its precise formulation can be a bit elusive. Leibniz characterizes the difference in various ways, each of which brings out certain features. Here is a basic formulation:

> *Whole:* A collection of parts that can be treated as a single thing.
>
> *Fictional Whole:* A collection of parts that can be treated only as many things.[29]

Two notes about this distinction: first, this is a distinction between *wholes*, which is to say collections of things with merely accidental unity. When I characterize wholes as collections that can be treated as a single thing, the sense of "single thing" here is not the sense of *being really one* or *being a true entity*. I will return to this point below. Second, Leibniz's own terminology varies, but I think that from the context it is clear that the following equivalences hold:

[29] "Fictional whole" is not Leibniz's terminology, though this terminology is grounded in what Leibniz says about wholes of this sort, and it provides a nice analog to his treatment of infinitesimals as "fictions". For discussion and terminology relating to Leibniz's view of infinitesimals, see Goldenbaum & Jesseph (2008). As far as I know, the use of the term "fictional whole" with respect to infinite wholes in Leibniz is due to Richard Arthur. See, e.g., Arthur (2011), 93.

Whole	*Fictional Whole*
= one whole	= multitude/complex
= collective whole	= distributive whole
= finitely many parts	= infinitely many parts

The cluster of terms Leibniz uses to capture this distinction indicates what my formulation attempts to capture: some collections can somehow be *one thing* while others are strictly multitudes. This is not an unfamiliar distinction in the post-Leibniz history of mathematics. It is what drives Cantor's distinction between sets and inconsistent multiplicities, and von Neumann's distinction between sets and proper classes: the intuition is that for whatever reason (typically it has to do with size), some collections cannot be *one thing*.[30]

Why is it for Leibniz that some collections of parts can be referred to *as a single thing* while others can be referred to *only as many things*? His answer is that some collections must be treated *as many things* because they have no quantity, strictly speaking, and so are not really wholes at all.

Consider the letter to Bernoulli, which we have already seen above.

> I concede an infinite multitude, but this multitude forms [*facit*] neither a number nor one whole [*unum totum*]. It only means that there are more elements than can be designated by a number, just as there is a multitude or complex of all numbers; but this multitude is neither a number nor one whole. (GM III, 575; LoC lxiii)

In Leibniz's view, when there are infinitely many elements in a collection, that collection is not strictly speaking *one thing*. The use of "one" here does not signal any sort of unity but rather indicates that though certain collections can be treated *as wholes*, infinite multitudes are not like this. Infinite multitudes remain multitudes (or complexes).

[30] For discussion of the relation between size and the set-theoretical paradoxes, see Hallett (1986).

The ability to treat a collection as *one thing* is connected, in Leibniz's view, to whether the collection has a determinate number of parts. In order to treat the collection as a single thing, it must have some specific quantity. But, in light of the impossibility of infinite number, a collection with infinitely many parts does not have a quantity, strictly speaking, since its quantity would be greater than any finite number. Leibniz makes this point in a letter to Des Bosses:

> I maintain, strictly speaking, that an infinite composed from parts [*ex partibus constans*] is neither one nor a whole [*neque unum esse, neque totum*], and it is not conceived as a quantity except through a fiction of the mind [*nec nisi per fictionem mentis concipi ut quantitatem*]. The indivisible infinite alone is one, but it is not a whole; that infinite is God. (LDB 53)

In this passage, Leibniz makes it explicit that a collection of infinitely many parts does not have quantity. He also adds something important: such a collection can be conceived as a quantity in a certain sense, namely through a fiction of the mind. So there is a sense in which we can fabricate it as a quantity through a mental fiction. For this reason, I am calling such a collection a *fictional whole*.

How does this fabrication work? Some help is found by looking to Leibniz's account of infinite series, since these are wholes in some sense, but consist of infinitely many terms. Consider Leibniz's remarks about the infinite series known as *Leibniz's Series*:

> even though...the series is produced to infinity, nevertheless, since it consists of one law of progression [*una lege progressionis constat*], the whole is sufficiently perceived by the mind [*tota satis mente percipitur*]" (GM V, 120; trans. Arthur 1998, 31).

The notion of a "law of progression" explicated in this passage in terms of infinite series can be generalized. It is, essentially, a description that picks out all and only the members of a particular collection.[31] Thus, for example, we can use the description *being a term in Leibniz's Series* to

[31] Leibniz explicitly claims that there is nothing answering to the designation "all the terms in the series", which he concludes on the basis that an infinite series has no last term": "...in fact there is no last term of the series, since it is unbounded.... Therefore we conclude finally that there is no infinite multiplicity, from which it will follow that there is not an infinity of things, either. Or it must be said that an infinity of things is not one whole [*unum totum*], i.e. that

pick out all and only the terms in Leibniz's Series. (Or we could use the formula $(-1)^n/(2n+1)$).

We can also use the description *being a Roman emperor* to pick out all and only the Roman emperors. In the latter case—that of the Roman emperors—the description picks out a whole. However, in the former case—that of being a term in Leibniz's Series—the description picks out only a fictional whole.[32]

In both cases—finite and infinite collections of parts—the whole is the product of a mental act: the difference is that in one case, we conceive of a genuine whole, while in the other, merely of a fictional one. This accounts for the sense in which, for example, the terms of Leibniz's Series are conceived as a quantity only through a fiction of the mind. The same mental act—considering things that answer to a certain description—that often leads us to wholes, leads us, in some cases, to merely fictional wholes. In these cases, there is, strictly speaking, no whole that answers to the description; there is merely a collection of parts that remains *many*. This is because, since the collection has infinitely many parts, there is, strictly speaking, no such thing as the collection of all the parts.[33]

There is an additional way Leibniz characterizes the distinction between wholes and fictional wholes. He also draws the distinction in terms of the *type of reference* that is possible in each case:[34]

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there is no aggregate of them" (A VI 4, 504; LoC 100-101). Leibniz's language in this passage is strong: there are no infinite multiplicities, i.e. they do not exist. Nevertheless, as the previous passage shows, we can perceive the whole series (in some sense) by grasping the law of progression.

[32] It is worth pausing to note that Leibniz's uses the notion of a "law of the series" to characterize the activity and unity of substances (see, for example LDV 455; AG 72). This might cause some worry, since the notion of a law is being connected to the characterization of a fictional whole here. However, the contexts in which Leibniz makes these claims are very different, as is the type of "series" under consideration.

[33] Leibniz's view on this point prefigures, in a sense, the restrictions of the so-called comprehension axiom of naïve set theory in order to avoid certain set-theoretical paradoxes, Russell's Paradox being one example. Leibniz's idea is that not all predicates pick out a genuine whole. A similar insight is employed by Cantor in his notion of *inconsistent multiplicities* and von Neumann in his notion of *proper classes*. See Cantor (1899), 114; von Neumann (1925). For discussion of von Neumann's view, see Hallett (1986), ch. 8.

[34] Arthur (2001), 110 develops the distinction between distributive and collective wholes. Arthur rightly notes that both the world and an organic body are distributive wholes, but given the context in which he is considering the

> There is also an actual infinite in the sense of a distributive whole [*totius distributivi*] but not a collective one [*collectivi*]. Thus, something can be stated [*enuntari potest*] about all numbers, though not collectively. In this way it can be said that for every even number there is a corresponding odd number, and vice versa; but it is not therefore accurately said that there is an equal multitude of even and odd numbers. (Leibniz to Des Bosses 1 September 1706; LDB 53)

Distributive wholes are what I have called "fictional wholes" and collective wholes are what I have called "wholes". Leibniz draws the distinction in terms of what can be accurately said of each type of whole, or perhaps better *how* something can be said about each type. In distributive wholes, though something can be stated of each part individually, it is not possible to say something about *all the parts together*, at least not accurately. Collective wholes, by contrast, can be referred to *as wholes*.[35]

It is tempting to think that Leibniz is making a general point about part-whole predication here. For example, we might take his point to be that in some cases (distributive wholes) but not in others (collective wholes), when we attempt to predicate something of the whole, this reduces to a predication of each part individually. But this is not the case. Leibniz is clear that all predication of wholes is reducible to predication of their parts individually. Consider the following:

> It is worth investigating in what way an entity through aggregation, such as an army or even a disorganized multitude of men, is one; and in what way its unity and reality differ from the unity and reality of a man. It seems that the chief difference is to be observed in their attributes and operations. Some attributes are said equally of the whole as of its parts, as, for example, that the army is located in the fields of Marathon, which is true of each individual soldier. Other attributes can be said only of the whole, as, for example, that the army is 30,000 strong, and that it is disposed in a lunar-shaped battle line. Nevertheless, all these things can be stated and expressed even if the multitude is not viewed as a single entity. Thus, I can say that 30,000 soldiers are present and that one soldier is situated with respect to another just as the battle line mentioned requires, so that certain ones are distanced from a fixed point by so much, others by so much. (A VI 4, 555-556; trans. Sleigh [1990], 123)

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problem (that of the world soul), the distinction between distributive and collective wholes doesn't provide a solution and so Arthur does not pursue it further, as I do, and connect it to other texts.

[35] Although, importantly, Leibniz does not treat a collective whole as an entity over and above its parts.

The point, then, is that all reference to wholes is reducible to reference to their parts. This is just as true of wholes as it is of fictional wholes. Even though wholes admit of collective reference, the information contained in such acts of reference is ultimately reducible to reference only to the parts. The important contrast in this regard is between wholes on the one hand and *substances* on the other. With substances, i.e. true beings, predication cannot be reduced to predication of their parts.[36] In terms of predication, then, we have a three-fold distinction:

True beings	**Wholes**	**Fictional Wholes**
Predication of entity *is not* reducible to predication of parts.	Predication of whole *is* reducible to predication of the parts	

For these reasons, it is more accurate to construe the distinction between collective and distributive reference as pertaining to *predications of quantity only*. That is to say, in a fictional whole, if we try to say something about the quantity of the whole, we cannot do so directly, since there is, strictly speaking, no quantity. But we can speak *as though* it has a quantity by means of distributive reference. This allows us to say the sorts of things we would like to say about various collections being set up in one-to-one correspondences without getting ourselves into problems such as the one indicated by Galileo's Paradox. On the other hand, if we try to refer to an infinite collection of parts collectively, we encounter paradox.

The notion of collective reference should not be confused with what we might call *irreducible reference*, which pertains only to substances. The point of Leibniz's distinction between collective and distributive reference is not that collective reference is irreducible

[36] For a contemporary view that shares a great deal with Leibniz on this point, see Merricks (2001).

reference, but simply that some wholes permit collective reference with respect to quantity, while others do not.[37]

The results so far can be summarized on the following table:

Type of Whole:	Fictional Whole	Whole
Number of Parts:	Infinitely Many (i.e. no determinate number)	Finitely Many (i.e. some specific number)
Predications of quantity:	Distributive	Collective
Quantity:	No quantity, strictly speaking, but can be fabricated via some description	Some finite quantity
Unity:	Accidental	Accidental

To reiterate, the distinction between wholes and fictional wholes is not concerned with considerations of unity. Both types of wholes have merely accidental unity. Furthermore, all of the texts in which Leibniz expresses his rejection of infinite number concern *wholes* (or a terminological variant). Thus, while Leibniz's rejection of infinite number certainly rules out infinite collections as *unities* it more importantly rules them out as *wholes*.

With this distinction in mind, it is natural to read Leibniz's expression of his rejection of infinite number in the passages above as a rejection of infinite wholes. The occurrence of the term "unity" (*unum*) in some of these passages—*unum totum*—can then be understood as designating a whole, i.e. a collection of parts that can be treated as a single thing (i.e. as *one whole*) without entailing a contradiction.

[37] There is an interesting connection between Leibniz's view and the recent developments in the logic of plurals. The linking question is this: must the subject of predication be singular or can we predicate of many things *as many*? Leibniz is, in a sense, not sensitive to this question, since in his view, when predicating of a collection, there is no single entity that is the subject of predication, but the predication *of the whole* is reducible to predication *to the parts individually*. Thus, although Leibniz does not require a singularizing device (e.g. a set or a mereological fusion) for predications of many things, he does not endorse full-blown plural predication either. For discussion of plural logic, see Yi (2005 and (2006), McKay (2006).

6. Composite Substances

I am now in a position to address the central question of the Specific Concern: do composite substances violate Leibniz's rejection of infinite number? Given the preceding analysis of wholes, in order to respond to this question I do not need to develop a relevant difference between the world-body and the body of a substance. It follows from what I have established above that composite substances, insofar as they are substances, will not be wholes at all. Recall that, in Leibniz's view, no true entity has a plurality of parts. Composite substances, if they are to be true entities, will not, therefore, have a plurality of parts. This opens up a very interesting and difficult set of questions concerning the relationship between a composite substance and its body. I cannot speak to all aspects of this question here, but I can say the following.

In virtue of having infinitely many parts, the body of a composite substance will be merely a multitude, not *one whole*. Similarly, it will not have a quantity, except fictionally, in virtue of the similarity between the mental act of considering the collection under a certain description and the act of considering a collection that really does have quantity. The fact that the form of the substance (or dominant monad) bears a unique relation to the substances constituting the body of the substance, such that the form-body composite is a composite substance, has no affect whatsoever on the quantity of the collection of things making up the body. The body of a composite substance is not a whole; it is merely a fictional whole. Even though it is joined to a form (or dominant monad), the body is actually infinitely divided and, therefore, has infinitely many parts. On this basis, the body is merely a fictional whole. Bodies are only multitudes and nothing more.

I expect this point to be granted without much resistance in the case of bodies without forms (or dominant monads). A slab of marble, for example, is not a substance, but a multitude of substances. What is less often noted in the literature is that it is also true of bodies *with* forms (or dominant monads). The bodies of composite substances are still only multitudes. A composite substance might be an *unum per se*, but as we have seen, this does not entail that its body is an *unum per se.* Leibniz is clear that the body of a composite substance is merely an *unum per accidens*. It is, in this respect, no different from a body without a form. A similar case can be made with respect to *wholeness*. A body without a form is merely a fictional whole; this is also true of bodies with forms. [38]

Importantly, this is explicitly Leibniz's view during his so-called Middle Years.[39] Even in the correspondence with Arnauld, where Leibniz is comfortable speaking of "corporeal substances", he is clear that the bodies of these substances are merely multiplicities. Further, in the passage from *Definitiones* (1685), which we have already considered above, Leibniz is clear that substances do not have parts. I think it is justifiable, therefore, to conclude that bodies are merely fictional wholes for Leibniz both in the Middle Years and in the Mature Period.

As to why the presence of a soul does not entail infinite number, recall that the difference between wholes and fictional wholes is that wholes have a determinate number of parts, and thus a quantity, while fictional wholes do not. Being united with a form (or dominant monad) does

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[38] It should be noted that, although Arthur (2001) argues that bodies are arithmetical unities, and thus *wholes*, since then Arthur has changed his view. See Arthur (2011), in which he develops a view much like the one I am arguing for here: bodies, even the bodies of composite substances are not, in his terminology, "true wholes".

[39] The designation "Middle Years" originates in Garber (1985) and indicates the years from roughly 1680-1700. There is an ongoing controversy surrounding Leibniz's commitment to corporeal substances during this period. The result above indicates that, at least with respect to whether a substance has parts, Leibniz has a consistent view from the 1680s until the end of his life, which means that if corporeal substances are to be mapped into Leibniz's other Middle-Years commitments, they have to be understood as *having no parts*.

not affect the number of parts that a body has. Therefore, it cannot affect the type of whole that a body is.

This is a surprising result in certain respects. A body certainly *seems* to be more of a whole than, for example, the collection of Roman Emperors. As I would have it, a body is merely a fictional whole, while spatio-temporally disjoint collections, such as that of the Roman Emperors, are, strictly speaking, wholes. But recall that, as I have argued, the designation "whole" is based on whether a collection of parts has a quantity. The collection of Roman Emperors, having only finitely many parts, has a quantity; a body, even the body of a composite substance, having infinitely many parts, does not. The distinction between wholes and fictional wholes tracks a specific feature of the collection of parts that make up a whole, namely whether there is or is not a determinate number of them. The presence of a form (or dominant monad) can do nothing to change this.

Another way to dull this apparently counterintuitive result is to note that wholes are by their very definition *constructed by minds*. A whole is characterized as a collection of things *considered together*. Nothing that depends on a perceiving mind can qualify as a true being for Leibniz. Thus, to be more of a whole is in no sense to be more of a being.

Nevertheless, we might ask: how does a composite substance differ from an aggregate, if its body is equally *fictional*? Crucially, the presence of a form (or dominant monad) does not *respond* to any feature of the body. Both the bodies of composite substances and aggregates are infinite multiplicities. The presence of a form does, however, change the way in which the substance depends on the parts of its body: it no longer depends on them the way an aggregate depends on its parts. In light of Leibniz's characterization of *parts* as *requisites*,[40] i.e. things

[40] See G II, 120 and GM VII, 18.

required by the wholes of which they are parts, a composite substance *has no parts*, because none of the parts of its body are, strictly speaking, required by the substance.[41]

7. Conclusion

In order to arrive at these conclusions, I have divorced the Specific Concern from the context in which it was originally formulated, namely Leibniz's argument against the world soul. I have done this because, as I have argued, I think that investigating the Specific Concern in this context has led other scholars to misconstrue the nature of Leibniz's rejection of infinite number. Now that, in my view, we have achieved some clarity about the nature of Leibniz's rejection of infinite number, I will conclude with some remarks concerning the implications of this result for our understanding of Leibniz's argument against the world soul, though I will not develop a complete interpretation of Leibniz's argument against the world soul here.

One significant implication of my argument is that Leibniz's argument against the world soul is not structured in the way that previous commentators have supposed. Leibniz's rejection of infinite number concerns *wholeness* and there is no basis for distinguishing an organic body from the world in terms of wholeness: both are fictional wholes, multitudes with infinitely many parts. (Even if we follow previous commentators and understand Leibniz's rejection of infinite number in terms of unity, there is no textually grounded basis on which to distinguish the unity of the world from the unity of an organic body: both are accidental unities.)

Of all the passages in which we find Leibniz's argument against the world soul together with his rejection of infinite number, only one appears to argue that the world cannot have a soul

[41] This point deserves to be developed at greater length, though doing so here would take me to far afield.

because it would have an infinite body, and that such a body cannot be *one thing*.[42] This type of argument certainly appears to lead to the question driving the Specific Concern. More often, however, Leibniz mentions his rejection of infinite number alongside his argument against the world soul, though his argument does not obviously rely on it. This suggests that Leibniz's argument against the world soul does not involve his rejection of infinite number in the way that other commentators have proposed. If correct, this suggestion would fit nicely with the results I have arrived at above.

Sorting out in detail Leibniz's argument against the world soul is, however, a separate endeavour and must be left for another occasion. For now, the conclusion I draw is that, at least with respect to his theory of composite substance, Leibniz the mathematician is not out of step with Leibniz the metaphysician.

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[42] See A VI 4, 1509; Loc 287.

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